

# Impartial pricing approach in double auction transactive distribution systems

Mehdi Jalali, Kazem Zare<sup>\*</sup>, Sajjad Tohidi

Faculty of Electrical and Computer Engineering, University of Tabriz, Iran

## ARTICLE INFO

### Keywords:

Transactive distribution system  
Micro-grid  
Distributed optimization  
Pricing strategy

## ABSTRACT

This paper introduces a new trading mechanism for future transactive distribution systems with multiple private microgrids. An independent economic entity as a third-party agent is designed to facilitate the secure information sharing and providing fair market among participants in the transactive system. Proposed trading mechanism is based on double auction market clearing which shares benefits of each transaction between corresponding seller and buyer. To ensure the network's technical constraints, real time operation of system is distributed between the players according to alternating direction method of multipliers (ADMM). The presented model satisfies complicated constraints and determines optimal scheduling. The proposed methodology is evaluated by implementing real-time operation on a modified 123-bus test distribution system with five micro-grids. Numerical studies show the beneficial properties of the proposed method.

## 1. Introduction

Extending distributed energy resources propels the structure of traditional distribution networks toward distribution systems. Furthermore, as a micro-grid, group of consumers can obtain more reliable electricity and more economic benefits by equipping with distributed generation units and control elements. CIGRE WG C6.21 defined MG as controllable systems including loads and DERs that can be operated in a coordinated way either connected to the network or islanded mode [1]. In the future distribution systems, there will be multiple private MGs that are responsible for providing their own consumptions. Therefore, designing an energy trading mechanism among MGs and distribution system is required to realize the energy exchange according to the agreement between the sellers and buyers. In this regard, transactive energy system approach can accommodate dynamic coordination between buyers and sellers in the distribution system [2]. In TES, each agent sends an economic value as a signal to the market agent that represents its favorite buying/ selling prices and corresponding quantities.

Generally, two central and layered decentralized optimization visions for the operation of distribution systems are introduced in [3]. The central approach is based on the extension of wholesale power market structure, where the operator is considered as DSO. Considering central approaches, the operator needs detailed visibility into the all levels of

distribution system. On the second vision, management at any given level of system only needs the visibility to the interface points with the next levels and doesn't require the excess information. The main advantages of distributed structures are providing security in information sharing and scalability for the management of large systems. Some operation management models for distribution systems with private micro-grids are presented in literature which can be distinguished in terms of dependency on the objective function, management, and market mechanisms. An integrated mechanism for DERs by introducing virtual power plant (VPP) is presented in [4], where the VPP gathers DER's detailed information and participates in the DAM and RTM implementing a two-stage programming. In [5], DSO publishes incentive signals to participants according to a transactive trading mechanism. Distribution locational marginal prices (DLMP) as incentive signals are responded by participants. As signal sharing proceeds, the feasible trades between DSO and participants are determined. DSO in this approach has additional responsibilities in comparison to MGs. A leader role for DSO is emerged in [6] which handles the day-ahead transactive market in the distribution level. A distributed approach for energy sharing among several residential micro-grids is presented in [7]. The optimal energy sharing problem in [7] is decomposed into a master problem and several sub-problems to tackle less computational burden and providing secure information sharing. Sub-problems are formulated to maximize utilized energy from solar PV arrays. Multi-follower bi-level programming (MFBP) approach is proposed in [8] to address the

<sup>\*</sup> Corresponding author.

E-mail address: [kazem.zare@tabrizu.ac.ir](mailto:kazem.zare@tabrizu.ac.ir) (K. Zare).

<https://doi.org/10.1016/j.ijepes.2021.107204>

Received 20 August 2020; Received in revised form 3 April 2021; Accepted 16 May 2021

Available online 12 September 2021

0142-0615/© 2021 Elsevier Ltd. All rights reserved.

## Nomenclature

### List of variables

DSO	Distribution system operator
MGO	Micro-grid owner
DS	Distribution system
DER	Distributed energy resources
MT	Micro-turbines
DAM	Day-ahead market
RTM	Real-time market
IEE	Independent economic entity
PS	Pricing strategy
TES	Transactive energy system.

### Indexes and sets

$t, k$	Time indexes
$i, j$	Indexes of Participants
$g$	Index of Microturbines
$b, r$	Index of Bus indexes
$\Omega^P$	Set of Participants
$\Omega^{MT}$	Set of Microturbines
$\Omega^{Bus}$	Bus indexes
$\kappa$	Iteration index
$\Phi_{i,b}^{SLK}$	Set of Slack buses in $i$
$\Phi_{i,b}^{MT}$	Set of MTs connected to $b$ in $i$
$\Phi_{i,b}^P$	Set of participants connected to $b$ in $i$
$\Phi_{i,b}^{BUS}$	Set of connected buses to $b$ in $i$ .

## Parameters

$\hat{\lambda}_{i,t}^{DA} / \hat{\lambda}_{i,t}^{RT}$	Forecasted price of DAM/RTM at time slot $t$ in $i$
$\hat{P}_{i,b,t}^d$	Forecasted demand in bus $b$ at time slot $t$ in participant $i$
$SU_i^g / SD_i^g$	Start-up/shut-down costs of MT $g$ in $i$
$UT_i^g / DT_i^g$	Minimum up/down times of MT $g$ in $i$
$P_{i,g}^{min} / P_{i,g}^{max}$	Minimum/maximum active power of MT $g$ in $i$
$Q_{i,g}^{min} / Q_{i,g}^{max}$	Minimum/maximum reactive power of MT $g$ in $i$
$a_i^g, b_i^g$	Cost coefficients of MT $g$ in $i$
$G_{br} / B_{br}$	Real/imaginary part of component in admittance matrix
$SL_{i,max}^{br}$	Maximum apparent power of line between bus $b$ and bus $r$ in $i$
$V^{max} / V^{min}$	Maximum/minimum limits of voltage magnitude.

## Variables

$PL_{i,t}^{br}, QL_{i,t}^{br}$	Powers flow in line between buses $b$ and $r$ in $i$ at $t$
$SL_{i,t}^{br}$	Apparent power flow in line between bus $b$ and bus $r$
$P_{i,b,t}^{inj} / Q_{i,b,t}^{inj}$	Injected active/reactive powers to bus $b$
$\delta_{i,t}^g$	Binary status variable of micro-turbine $g$ in $i$ at $t$
$SUC_{i,t}^g$	Start-up cost of micro-turbine $g$
$SDC_{i,t}^g$	Shut-down cost of micro-turbine $g$
$P_{i,t}^g / Q_{i,t}^g$	Active/reactive powers of MT $g$
$P_{i,t}^{DA} / P_{i,t}^{RT}$	Active power transaction with DAM/RTM in $i$ at $t$
$Q_{i,t}^G$	Injected reactive power from upper grid in $i$ at $t$
$V_{i,t}^b / \theta_{i,t}^b$	Voltage magnitude/angle of bus $b$ in $i$ at $t$ .

optimal operation of distribution systems with multiple micro-grids. Due to using single level KKT equivalent of MFBB, the proposed method in [8] doesn't fit for the cases with independent micro-grids [8].

A distributed economic model for coordinated energy management of a multi micro-grid system is presented in [9]. In [10], from the DSO's standpoint, distributed approach for the operation of networked MGs is presented. A two-level structure based on system of systems methods for the distribution companies and micro-grids is presented which determines bilateral contracts between entities [11]. Another distributed energy management method for a micro-grid consisting of multiple DERs and controllable loads are studied in [12,13]. In [14,15] the alternating direction method of multiplier (ADMM) is implemented to relax complicating power balance between the distribution system operator and micro-grids. In fact, the result of this approach corresponds to the optimal cost of DSO and MGs. Obviously, the results of [13,15] wouldn't lead to the Nash equilibrium point. In [16], flexible activation on distribution systems with three types of players are studied. The players (Distribution System Operator (DSO), aggregator and residential end-users) are considered in a specific layer. To solve the system operation problems over the day-ahead operating vision in distributed manner, virtual prices are computed and exchanged between the players based on dual decomposition. A peer-to-peer (P2P) energy trading mechanism has been introduced in [17,18] which handles energy exchange between individual players (peers) without any intervention of an intermediary. The main challenge in the P2P approaches is ensuring against the violation in the network technical constraints. In [18], all peers contribute in continuous double auction (CDA) market mechanism and CDA is run for each time slot separately. Optimizing the utility preferences by using real time and forward contracts in P2P approach has been addressed in [17]. Furthermore, price adjustment process for the forward and real time is driven in iterative manner between the peers. Same authors used P2P framework for energy trading within an energy system with multi-class prosumers in [19]. An un-scalable balancing market framework is presented in [20] which adopts the

concept of network constrained transactive energy for facilitating interactions between the transmission system operator and aggregated prosumers. Nested transactive market inspiring resiliency in trading has been structured in [21], which facilitates exchanges in terms of peer-to-peer, peer to markets and markets to markets. The presented approach in [21] is limited to handle transactions between the peers of each nested community. A market based trading approach named as energy collective is presented in [22] where the market operator allows prosumers to share excess energy within and outside the considered community. In [23], collaborative operation of networked microgrids inspiring TES concept is distributed between individual MGs to determine the optimal interaction with upper grid. For market based transactive energy coordination approaches, the influence of interaction limit and bids and offers of players on the performance of players utility and market efficiency have been addressed in [24]. In [24], feeder coordinator represents a double auction market for the players and determines clearing price based on collected bids and offers. The proposed mechanism in [25] is based on DLMP and bi-level programming, where DSO has the supervisor role. Furthermore, the clearing price is regarded as the interaction point of players' bids and offers. This point challenges the transparency and information security.

The main contribution of this paper is introducing an impartial market clearing mechanism for transactive energy systems. In comparison with reviewed papers, an independent economic entity (IEE) is used to facilitate secure information sharing which is not on the domination of DSO and MGOs. IEE will determine exchange power and corresponding price of each transaction only considering its own seller and buyer. The proposed structure doesn't allow any operator to access other participants' information and economic values. Moreover, fair benefit sharing between the sellers and buyers is achieved by using the proposed pricing strategy. Against common market clearing approach, the proposed strategy shares all benefits of each transaction between the corresponding seller and buyer. The physical power delivery among participants regarding the approved transactions and satisfaction of

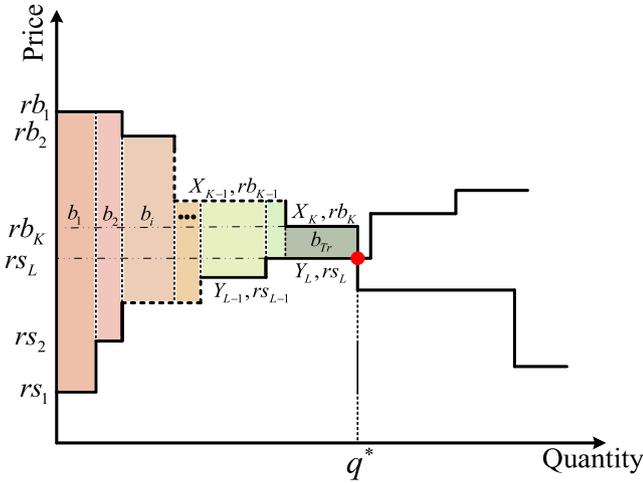


Fig. 1. The pricing structure of selling offer and buying bid.

technical constraints has been tackled by implementing ADMM. Implemented distributed optimization provides a framework for the satisfaction of voltage magnitudes and angles equality at the connection points. Furthermore, distributed optimization addresses optimal self-scheduling of energy resources in MGs and distribution system.

The rest of this paper is structured as follows. The proposed transactive system and double auction market clearing are described in Section 2. The real time scheduling of distribution system and MGs from their owners point of view is formulated in Section 3. Section 4 represents details and results of numerical analyses of proposed strategies over a test distribution system. Finally, Section 5 demonstrates the conclusion remarks.

## 2. Proposed methodology

In this section, mechanism of proposed transactive distribution system and market clearing strategy are represented in details.

### 2.1. Transactive distribution system

Electricity utilities and consumers are working on extending DERs and micro-grids to provide more reliable and resilient oriented energy systems. DSO, which has invested in the distribution network, is responsible for the maintaining, protection and securing the network. However, in the cooperation with MGs, there isn't any reason for MGOs to share their information with DSO. The proposed IEE is considered as an intermediary in exchanging economic values (i.e. selling offers and buying bids) between participants in the system (DSO and MGs). This mechanism does not allow DSOs to access micro-grids' information and it cannot impose DSO's volition on the micro-grids. We supposed that the participants provide economic values (i.e. buying bids ( $EV_B$ ) and selling offers ( $EV_S$ )) with respect to other participants and submit them to the IEE. The sorted economic values in the cases of buying and selling are:  $EV_B = \{(rb_1, X_1), \dots, (rb_j, X_j), \dots, (rb_m, X_m)\}$  and  $EV_S = \{(rs_1, Y_1), \dots, (rs_k, Y_k), \dots, (rs_n, Y_n)\}$ , where  $X_j$  and  $Y_k$  are quantities corresponding buying block  $j$  and selling block  $k$ , respectively.

For any possible transactions, IEE sorts the economic values in the ascendant price order under buying circumstances as (1), and in the descendent price order as (2) in selling cases:

$$rb_1 \geq rb_2 \geq rb_j \dots \geq rb_m \quad (1)$$

$$rs_1 \leq rs_2 \leq rs_k \dots \leq rs_n \quad (2)$$

The sorted economic values for the selling and buying are illustrated in Fig. 1. Referring to Fig. 1, the trading blocks can be determined based on

block matrix  $B[b_{jk}]$  with  $(n \times m)$  dimension. The parameter  $b_{jk}$  is 1, if  $rb_j \geq rs_k$ . All non-zero elements in block matrix  $B$  are considerable trading blocks  $N_T$ , which are ordered based on (1) and (2).

The implemented approach for quantification of interaction point  $q^*$  is presented in Algorithm 1. As demonstrated in Fig. 1, there are  $L$  buying and  $K$  selling blocks which are potentially candidate to be considered. In this paper, first pricing strategy (PS1) is corresponding to the price of critical point. Considering the interaction point  $q^*$ , IEE determines trading blocks ( $B = \{b_1, b_2, \dots, b_{Tr}\}$ ). The last trading block  $b_{Tr}$  ends at the interaction point  $q^*$ .

As shown in Fig. 1, trading block  $i$  will be considered as a potential energy exchange between the corresponding buyer and seller. The price associated to each trading block is proposed to be average of seller's offer and buyer's bid. This mechanism for impartial trading which shares benefit of each transaction between the corresponding seller and buyer has been presented in Algorithm 1.

**Algorithm 1.** Proposed impartial double auction market clearing for each transaction

---

```

1: Set  $q_0 = 0, q_0^* = 0, \alpha_0 = 0$  and  $\beta_0 = 0$ .
2: for Trading block  $i-1 : T_N$  do
3:   Trading block  $i$  belongs to the buying part  $j$  and selling part  $k$ .
4:   if  $rb_j \geq rs_k$  then do:
5:      $q_i = \min(X_j - \alpha_{i-1} \times q_{i-1}, Y_k - \beta_{i-1} \times q_{i-1})$ 
6:     if  $q_i < (X_j - \alpha_{i-1} \times q_{i-1})$  then  $\alpha_i = 1$ . Else:  $\alpha_i = 0$ .
7:   end if
8:   if  $q_i < (Y_k - \beta_{i-1} \times q_{i-1})$  then  $\beta_i = 1$ . Else:  $\beta_i = 0$ .
9:   end if
10:   $\pi_i = (rb_j + rs_k)/2$ .
11:   $q_i^* = q_{i-1}^* + q_i$ 
12: end if
13: end for
14: Return quantity and price of trading blocks  $(q_i, \pi_i)$ .
```

---

In Algorithm 1, within an iterative manner which starts from the first trading block, the quantity of energy exchange between the seller and buyer in trading block  $i$  is determined according to (3). Here, two auxiliary binary parameters  $\alpha_i$  and  $\beta_i$  are used to facilitate the calculation process. If whole the buying part  $j$  (selling part  $k$ ) is considered in trading block  $i$ , then  $\alpha_i$  ( $\beta_i$ ) will be equal to 0 otherwise, it will be 1. For the first block, the values of  $\alpha_0$ ,  $\beta_0$  and  $q_0^*$  are initialized to 0. To share the benefit of each trading block between seller and buyer, the associated power and price for trading block  $i$  can be calculated as:

$$q_i = \min(X_j - \alpha_{i-1} \times q_{i-1}, Y_k - \beta_{i-1} \times q_{i-1}) \quad (3)$$

$$\pi_i = (rb_j + rs_k)/2 \quad (4)$$

Through the Algorithm 1, the value of interaction point can be determined as follow:

$$q_i^* = q_{i-1}^* + q_i \quad (5)$$

In the proposed transactive architecture, despite the approach of [11,13], DSO and MGOs participate in the transactions which are profitable for both of them. To satisfy the secure information sharing and equality in terms of voltage magnitudes and angles, the trading mechanism and physical interactions have been distinguished.

The proposed mechanism avoids undermining inactivation of participants. In this regard, suppose if a buyer publishes its own bids less than its normal prices to IEE, this buyer might be removed from the list of final buyers regarding (1). This problem can be occurred for the sellers if they offer in higher price than normal offers regarding (2). Furthermore, IEE can prevent the participation of any sellers and buyers with abnormal bids.

The proposed distributed optimization and energy delivery of the approved transactions are addressed in the next sections.

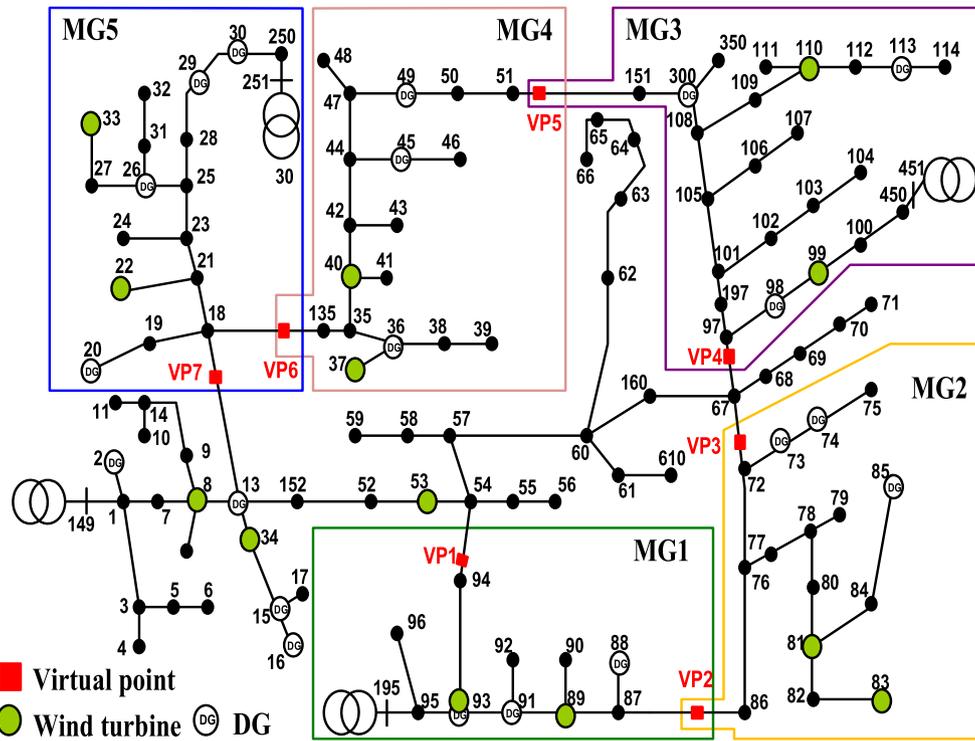


Fig. 2. Single line of modified 123 bus IEEE test system.

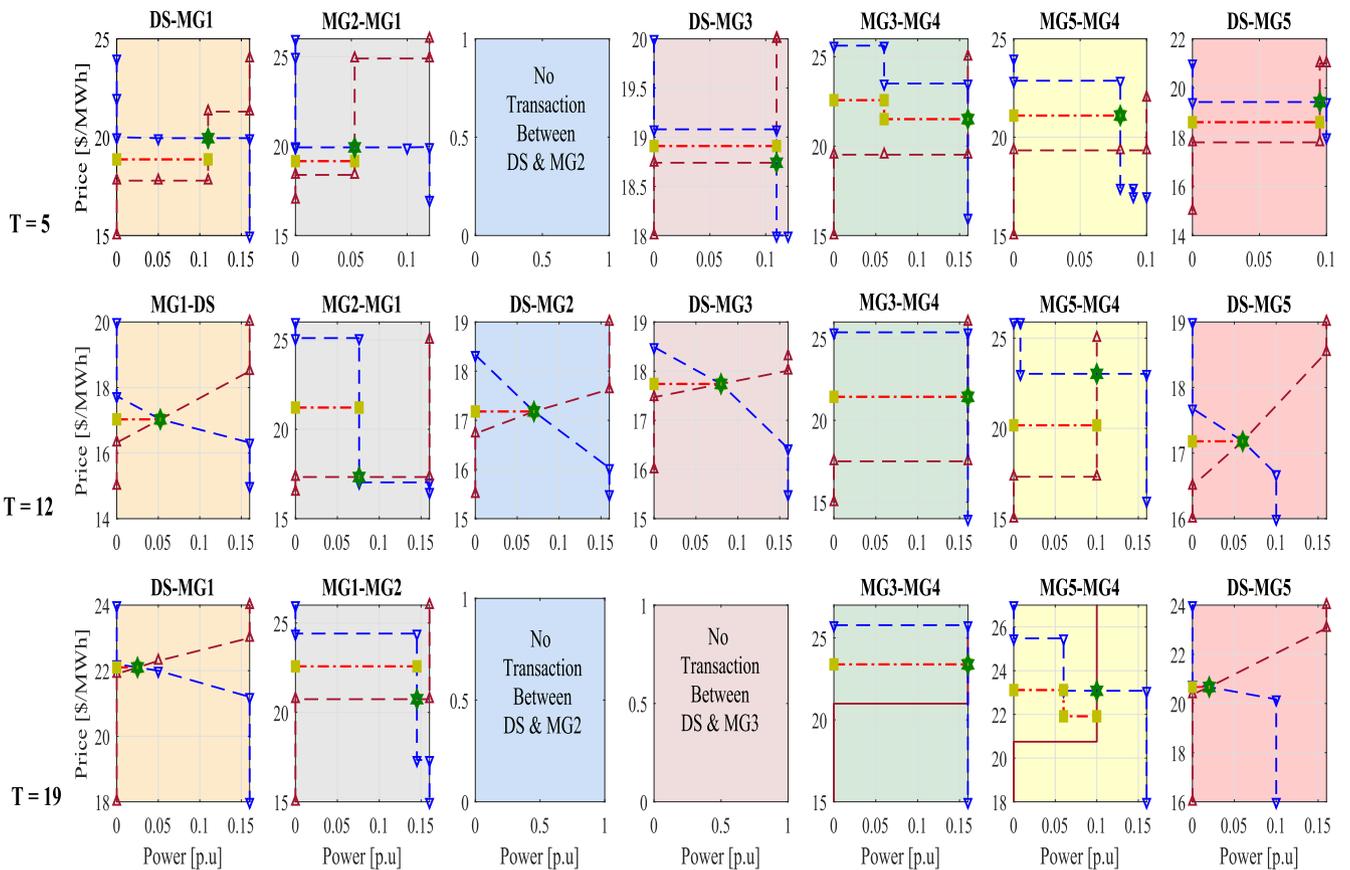


Fig. 3. Transaction clearing by IEE at time slots 5, 12 and 19. The corresponding Seller-Buyer is shown as a pair of entities above each box. The descending curves represent the seller's offer and the ascending curve indicates the buyer's bid. The dark green hexagons indicate transaction prices based on the first strategy and the red lines limited to olive squares representing the proposed pricing strategy.

**Table 1**  
Difference of entities' share from social welfare considering proposed and first pricing strategy.

	Pricing Strategy	DS	MG1	MG2	MG3	MG4	MG5
DS-MG1	PS1	1.26	0.77	-	-	-	-
	PS2	1.02	1.015	-	-	-	-
MG1-	PS1	-	4.74	31.98	-	-	-
	PS2	-	18.36	18.75	-	-	-
DS-MG2	PS1	0	-	0	-	-	-
	PS2	0	-	0	-	-	-
DS-MG3	PS1	2.12	-	-	0.47	-	-
	PS2	0.89	-	-	1.30	-	-
MG3-	PS1	-	-	-	51.42	67.61	-
	PS2	-	-	-	59.52	59.52	-
MG5-	PS1	-	-	-	-	18.32	25.77
	PS2	-	-	-	-	33.23	25.77
DS-MG5	PS1	5.56	-	-	-	-	2.34
	PS2	0.28	-	-	-	-	2.34
Total	PS1	8.95	5.51	31.99	51.90	85.93	28.11
	PS2	2.18	19.38	18.75	60.82	92.75	28.11

### 2.2. Physical power exchange

As presented in Algorithm 1, each player in the transactive distribution system submits its own economic value as bid/offer to the IEE and, IEE clears feasible transactions based on their preference selling offers and buying bids. Then, the accepted transactions are sent back to the corresponding sellers/buyers.

### 3. Problem formulation

In this section, the self-scheduling of resources and physical energy exchange among entities are addressed.

#### 3.1. Energy management

It is supposed that the distribution system operator and microgrids have the opportunity to participate in the electricity markets such as DAM and RTM. According to the major electricity markets' rules such as CAISO, day-ahead market's gate will be closed at a specific time prior to the trading target horizon and after that time DAM's price will be published for all participants. In addition, the RTM opens at a specific time slot of the trading day and closes at a specific time before target

horizon and results are published prior to the trading interval. Since entities don't have any information about prices in the day-ahead and real-time markets before the clearing time, they need to forecast prices prior to the gates closing time. Therefore, the general form of considered objective function for the real-time operation problem of participant  $i$  (i.e. microgrids and distribution system) is formulated in a rolling horizon fashion as:

$$f_i(u_i) = \sum_{k=t-T}^t (\hat{\lambda}_{i,t|k}^{DA} P_{i,t|k}^{DA} + \hat{\lambda}_{i,t|k}^{RT} P_{i,t|k}^{RT}) + \sum_{k=t-T}^t \sum_{g \in \Omega_i^{MT}} (a_i^g P_{i,t|k}^g + b_i^g + SUC_{i,t|k}^g + SDC_{i,t|k}^g) \quad (6)$$

The first term in Eq. (6) denotes the cost of purchased power from DAM and RTM. Parameters  $\hat{\lambda}_{i,t|k}^{DA}$  and  $\hat{\lambda}_{i,t|k}^{RT}$  denote forecasted prices of day-ahead and real-time markets at time slot  $t$  for the  $k$ -hour ahead provided by participant  $i$ . The second term of (6) is used to model operation, start-up and shut-down costs of micro-turbines within participant  $i$ . Since IEE clears transactive trades before participating in the RTM, the cost and revenue of local transactions are excluded in (6).

The operating constraint and load flow equations in the distribution systems and micro-grid are expressed as:

$$SUC_{i,t|k}^g \geq SU_i^g (\delta_{i,t|k}^g - \delta_{i,t|k-1}^g) \quad \forall i \in \Omega^P, g \in \Omega_i^{MT}, \forall k = t : T \quad (7)$$

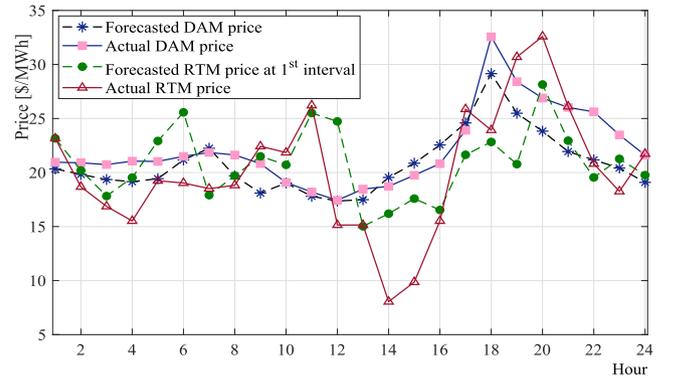


Fig. 5. Real time market and Day ahead market's prices.

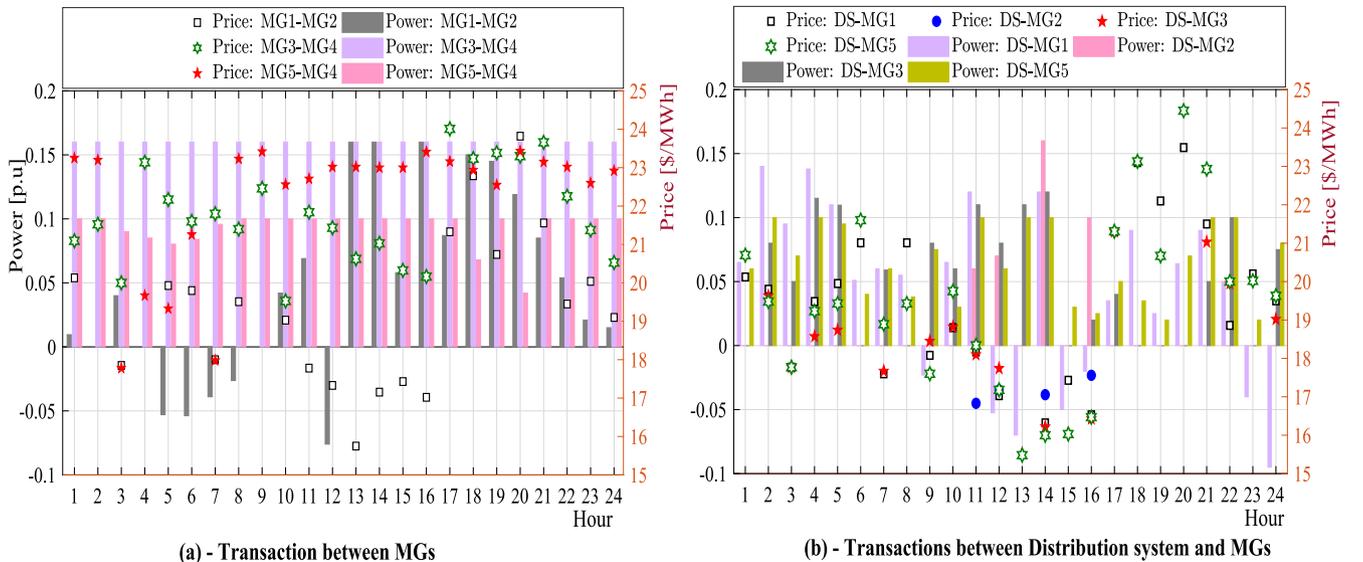


Fig. 4. Energy exchange at the operating horizon, (a)- Price and exchanged power between electricity markets and DSO, (b)- Bids of buyers and transactions between entities.

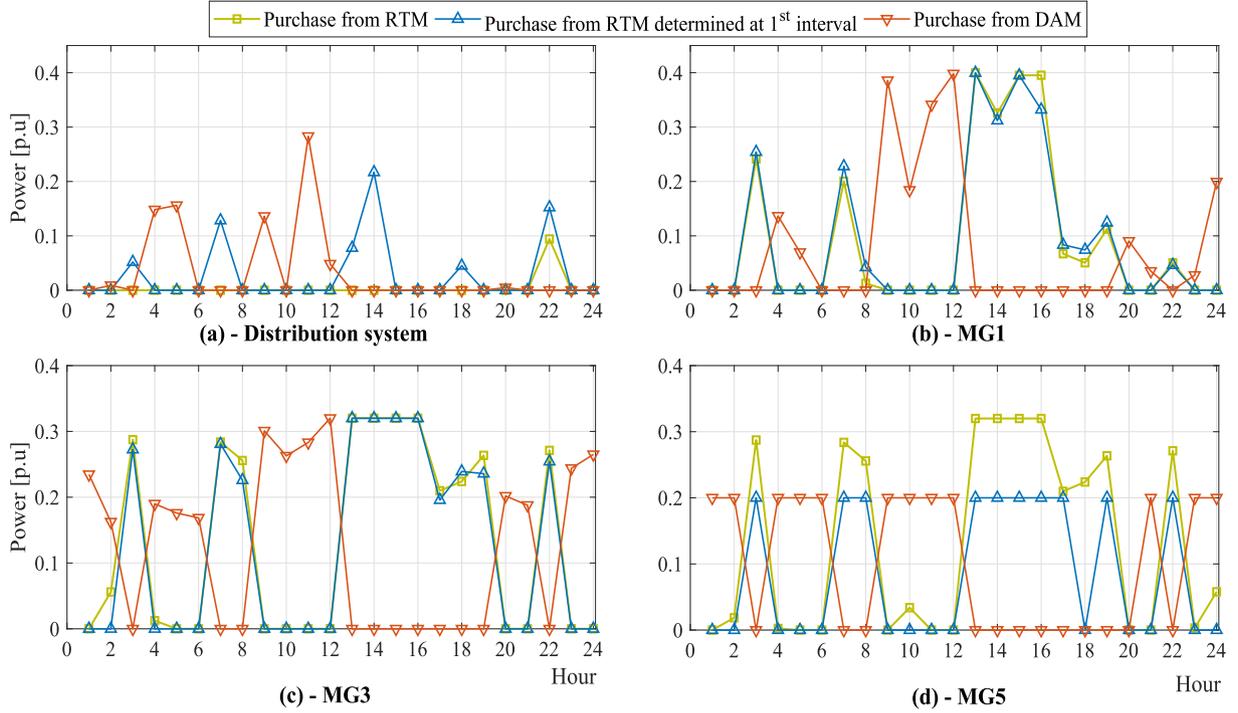


Fig. 6. Energy exchange in the operating horizon, (a)- Price and power exchange between electricity markets and DSO, (b)- Bids of buyers and transactions between entities.

$$SDC_{i,t,k}^g \geq SD_i^g (\delta_{i,t,k}^g - \delta_{i,t,k-1}^g) \quad \forall i \in \Omega^P, g \in \Omega_i^{MT}, \forall k = t : T \quad (8)$$

$$P_{g,i}^{\min} \times \delta_{i,t,k}^g \leq P_{i,t,k}^g \leq P_{i,g}^{\max} \times \delta_{i,t,k}^g \quad \forall i \in \Omega^P, g \in \Omega_i^{MT}, \forall k = t : T \quad (9)$$

$$Q_{g,i}^{\min} \times \delta_{i,t,k}^g \leq Q_{i,t,k}^g \leq Q_{i,g}^{\max} \times \delta_{i,t,k}^g \quad \forall i \in \Omega^P, g \in \Omega_i^{MT}, \forall k = t : T \quad (10)$$

$$\sum_{\Phi_{i,b}^{SLK}} (P_{i,t,k}^{DA} + P_{i,t,k}^{RT}) + \sum_{g \in \Phi_{i,b}^{MT}} P_{i,t,k}^g - \sum_{j \in \Phi_{i,b}^P} P_{i,j,t+1}^S + \sum_{j \in \Phi_{i,b}^B} P_{i,j,t+1}^B - \hat{P}_{i,b,t,k}^d = P_{i,b,t,k}^{inj} \quad \forall i \in \Omega^P, b \in \Omega_i^{Bus}, \forall k = t : T \quad (11)$$

$$\sum_{\Phi_{i,b}^{SLK}} Q_{i,t,k}^G + \sum_{g \in \Phi_{i,b}^{MT}} Q_{i,t,k}^g - \hat{Q}_{i,b,t,k}^d = Q_{i,b,t,k}^{inj} \quad \forall i \in \Omega^P, b \in \Omega_i^{Bus}, \forall k = t : T \quad (12)$$

$$(2V_{i,t,k}^b - 1)G_{bb}^i + \sum_{r \in \Phi_{i,b}^{BUS}} B_{br}^i (\theta_{i,t,k}^b - \theta_{r,t,k}^r) + G_{br}^i (V_{i,t,k}^b + V_{r,t,k}^r - 1) = P_{i,b,t,k}^{inj} \quad \forall i \in \Omega^P, b \in \Omega_i^{Bus}, \forall k = t : T \quad (13)$$

$$-(2V_{i,t,k}^b - 1)B_{bb}^i + \sum_{r \in \Phi_{i,b}^{BUS}} G_{br}^i (\theta_{i,t,k}^b - \theta_{r,t,k}^r) - B_{br}^i (V_{i,t,k}^b + V_{r,t,k}^r - 1) = Q_{i,b,t,k}^{inj} \quad \forall i \in \Omega^P, b \in \Omega_i^{Bus}, \forall k = t : T \quad (14)$$

$$PL_{i,t,k}^{br} = G_{br}^i (V_{i,t,k}^b - V_{r,t,k}^r) + B_{br}^i (\theta_{i,t,k}^b - \theta_{r,t,k}^r) \quad \forall i \in \Omega^P, r, b \in \Omega_i^{Bus}, \forall k = t : T \quad (15)$$

$$QL_{i,t,k}^{br} = B_{br}^i (V_{i,t,k}^b - V_{r,t,k}^r) + G_{br}^i (\theta_{i,t,k}^b - \theta_{r,t,k}^r) \quad \forall i \in \Omega^P, r, b \in \Omega_i^{Bus}, \forall k = t : T \quad (16)$$

$$SL_{br}^{i,t,k} \simeq PL_{i,t,k}^{br} + QL_{i,t,k}^{br} \cdot \zeta_{i,t,k}^{br} \quad \forall i \in \Omega^P, r, b \in \Omega_i^{Bus}, \forall k = t : T \quad (17)$$

$$-SL_{i,\max}^{br} \leq SL_{i,t,k}^{br} \leq SL_{i,\max}^{br} \quad \forall i \in \Omega^P, r, b \in \Omega_i^{Bus}, \forall k = t : T \quad (18)$$

$$V^{\min} \leq V_{i,t,k}^b \leq V^{\max} \quad \forall i \in \Omega^P, b \in \Omega_i^{Bus}, \forall k = t : T \quad (19)$$

The set of participant  $i$ 's independent decision making variables is defined as  $u_i = \{P_{i,t,k}^{DA}, P_{i,t,k}^{RT}, P_{i,t,k}^g, Q_{i,t,k}^g, \delta_{i,t,k}^g, V_{i,t,k}^b, \theta_{i,t,k}^b\}$ . The start-up and shut-down costs are formulated as (7) and (8), respectively. Constraints (9) and (10) satisfy the minimum and maximum active and reactive power generation of micro-turbines. The details of implemented load flow method can be followed in [27]. According to the load flow method presented in [27], the active and reactive power balances are modeled by using (11) and (12). The injected active and reactive powers into each node are equal to (13) and (14), respectively. The linearized form of active and reactive powers flow between nodes  $b$  and  $r$  are developed according to (15) and (16), respectively. By implementing Taylor series, the apparent power flowing through the line between nodes  $b$  and  $r$  is approximated by introducing an auxiliary parameter ( $\zeta_{i,t,k}^{br}$ ) in (17) and is limited to the line capacity in (18). The maximum and minimum limits of voltage magnitudes are shown in (19).

### 3.2. Distributed energy management

ADMM as a distributed optimization technique is implemented to distribute the operation scheduling and physical exchanging of the approved transactions [26]. The connection point of two entities corresponding to each transaction is modeled by a zero-impedance line between participants  $i$  and  $j$ . By integrating voltage magnitude and voltage angle in a vector as  $x_{ij} = [V_i^b, \theta_i^b]$ , it is obvious that at each time slot  $t$ , values of  $x_{ij}$  should be equal with  $x_{ji}$ :

$$x_{ij,t(t+1)} = x_{ji,t(t+1)} \quad : \quad \lambda_{ij,t(t+1)} \quad (20)$$

The dual parameters corresponding to the complicated constraint (20) are regarded as  $\lambda_{ij} = [\lambda_{ij}^V, \lambda_{ij}^\theta]$ .

Regarding [28], the general form of implemented ADMM for relaxing complicated constraint (20) between individual entities is followed in an iterative manner, as presented in Algorithm 2. In (20),  $\rho_{ij}$  is a

positive vector corresponding to equality of voltage magnitudes and angles in connection points as  $[\rho_{ij}^V, \rho_{ij}^\theta]$ . It is noteworthy that the transactions between entities are predetermined by IEE. Distributed model based on relaxing complicated constraint (20) is stated using Augmented Lagrangian function form as:

#### Algorithm 2. Proposed physical power exchange and self-scheduling

---

**Procedure:** Independent economic entity send approved trading blocks to the participants.  
 2: Participant  $i$  gathers its own trading blocks and act as follow:  
**Set iteration**  $\kappa = 1$ ,  $\lambda_{ij}^\kappa$ ,  $x_{ij}^\kappa$ ,  $\rho_{ij} > 0$ .  
 4: **while** Considering termination criteria  
     **for**  $i \in \Omega^P$  **do**  
         6:  $x_{ij}^\kappa \in \arg \min_{u_i} f_i(u_i) + \sum_{j \in \Omega_i^k} \lambda_{ij}^\kappa (x_{ij}^\kappa - x_{ji}^\kappa) + \sum_{j \in \Omega_i^k} 0.5 \rho_{ij} \|x_{ij}^\kappa - x_{ji}^\kappa\|_2^2$   
         s.t: constraints (7) – (19).  
     **end for**  
 8:  $\lambda_{ij}^{\kappa+1} = \lambda_{ij}^\kappa + \rho_{ij} \times (x_{ij}^{\kappa+1} - x_{ji}^{\kappa+1})$   
      $\kappa \leftarrow \kappa + 1$   
 10: **end while**  
**Return**  $x_{ij}, u_i$

---

$$\min_{u_i} f_i(u_i) + \sum_{j \in \Omega_i^k} \lambda_{ij,t(t+1)}^\kappa (x_{ij,t(t+1)}^\kappa - x_{ji,t(t+1)}^\kappa) + \sum_{j \in \Omega_i^k} 0.5 \rho_{ij} \|x_{ij,t(t+1)}^\kappa - x_{ji,t(t+1)}^\kappa\|_2^2$$

$$s.t: \text{ constraints (7) – (19).} \quad (21)$$

Distributed model (21) according to Algorithm 2, subjected to constraints (7)–(19) are minimized by DSO and MGOs at each time  $t$ , before real-time market gate closing time and after publication of IEE. The first term of (21) is presented in (6). Dual variables of constraint (20) are updated at iteration  $\kappa + 1$  and time slot  $t$  as:

$$\lambda_{ij,t(t+1)}^{\kappa+1} = \lambda_{ij,t(t+1)}^\kappa + \rho_{ij} \times (x_{ij,t(t+1)}^{\kappa+1} - x_{ji,t(t+1)}^{\kappa+1}) \quad (22)$$

## 4. Numerical studies and result analysis

The case studies are conducted on modified single phase 123-bus IEEE test distribution system. Multiple microgrids are considered as distinct parts of the test system. Single line diagram of considered test system is illustrated in Fig. 2. As shown in Fig. 2, the connection points between individual MGs and the distribution system are demonstrated as virtual points. Detailed information including forecasted prices, electricity loads and wind turbine output powers in the MGs and distribution system are presented in [29]. Moreover, [29] includes the data of energy resources, microgrid and network topology data. All the studies are conducted via linked MATLAB 2019b and GAMS 28.3. The effectiveness of presented methodology is evaluated in three sections as follows:

### 4.1. Analysis of proposed pricing strategy

As described in Algorithm 1, IEE receives the entities submitted selling offers and buying bids and then, determines the accepted associated price as well as trading blocks between entities that are connected physically to each other. This mechanism of transactions between entities for time slots  $t = 5$ ,  $t = 12$  and  $t = 19$  are illustrated in Fig. 3. Fig. 3 includes sellers' offer, buyers' bid and the associated prices according to two pricing strategies. The pair of seller-buyer is indicated on top of each transaction in Fig. 3. As described in the previous section, two strategies are assigned for trading blocks. In the first pricing strategy (PS1), the price of approved transaction is equal to the price corresponding to the interaction point of integrated seller's offer and buyer' bid. The assigned price of this case is known as the market clearing price. In the second (proposed) pricing strategy, IEE shares the benefits of each transaction

between the corresponding seller and buyer by setting the average of offer and bid as the accepted price. As shown in Fig. 3, the price associated with the first and second case studies are plotted in dark green hexagons and red dash lines, respectively.

Regarding Fig. 3, by using the first strategy, the transaction between distribution system and MG1 is cleared on the suggested value of buyer (MG1) and the distribution system sells more than its offer at time slot  $t = 5$ . Hence, the buyer does not obtain any benefit from the available social welfare. At time slot  $t = 12$ , the IEE clears trade between MG1 and MG2 on the seller's offer (MG1), and the buyer (distribution system) earns total social welfare benefits. By implementing the proposed strategy, the price of each transaction is considered as the average of seller's offer and buyer's bid. Thereupon, the benefit of each transaction is shared between the corresponding participants. For instance, the selling offers and buying bids of MG3 and MG4 at time slot  $t = 5$  and also, MG5 and MG4 at time slots 5, and 19 create some trading blocks. Based on the proposed pricing strategy, which is described in Algorithm II, an average price is assigned for each trading block. Therefore, the benefits of transactions considering trading blocks are shared between the seller and buyer. This fact is illustrated in Table 1. As indicated in Table 1, for each transaction, the portion of seller and buyer is similar by using proposed pricing strategy and the first approach (PS1). Furthermore, by using the proposed pricing strategy, the benefit of each entity from the social welfare is modified. However, it can be concluded that the second strategy is fairer than the first one.

### 4.2. Trading analysis between entities

Entities provide selling and buying bids considering their pre-determined schedules from electricity markets. Distribution system and microgrids submit the selling and buying bids into IEE, as shown in Fig. 3. The approved trading quantity (power) and the associated price, according to interaction point of selling and buying curves, for each time interval between MG $i$  - MG $j$  and MG $i$  - distribution system are illustrated in Fig. 4 (a)-(b). The power between MG5 and MG4 are exchanged in the maximum capacity of connection line (0.16 p.u). Since MG4 doesn't have access to the upper grid and, its micro-turbines produce in high prices, it buys power from MG3 and MG5. In addition, due to double auction trading between MGs, the price of transactions through the hours is cleared in different values. At time slot  $t = 3$ , MG1 sells 0.035 p.u to MG2. However, MG1 is agreed with MG2 to buy 0.055 p.u at time slot  $t = 5$ . In comparison with trading price between entities, it can be figured out if the trading prices are less than real time markets, entities will not buy from RTM. For instance, at time slot  $t = 8$ , MG1 buys from MG2 and neglects buying from RTM.

### 4.3. Participating analysis in day-ahead and real-time markets

As shown in Fig. 2, distribution system, MG1, MG3 and MG5 are connected to the upper grid. By considering the restructured form of power system, it is assumed that entities can participate in day-ahead and real-time markets, following price taking strategies [30]. Independent system operator clears DAM at the specific hour in a day prior to the trading day and RTM in a specific hour or few minutes prior trading interval. Entities (DS and MGs) based on their prediction from the markets prices and their net consumption provide optimal schedules. The forecasted and cleared prices of DAM and RTM are plotted in Fig. 5. In Fig. 6 (a)-(d), the purchased power from DAM, the pre-determined power exchange with RTM determined before trading day and finalized purchased power from RTM are illustrated. By considering difference in forecasted and cleared prices of DAM and RTM in Fig. 5, it can be observed that the finalized power purchases from RTM are different from those determined at first interval. Furthermore, it can also be noticed that entities decide to purchase from DAM and RTM based on their price forecasts. For instance, at  $t = [13-19]$ , it is obvious from Fig. 6 that entities based on their predictions of the DAM and RTM prices

decide to not buy from DAM, because they forecasted that RTM price would be less than DAM price.

## 5. Conclusion

In this paper, real time trading mechanism for entities within the transactive distribution system is proposed. An independent economic entity is introduced to facilitate trading between the individual entities. Furthermore, a profit-sharing based pricing strategy between the sellers and buyers is presented. Moreover, real time scheduling by using distributed optimization is implemented, which ensures physical power exchange among entities by satisfying the technical constraints such as equality of voltages in connection points. Moreover, the performance of presented method is studied in the real-time operation of a test system by using updated forecasts of electricity markets' prices and net load consumption. The results show that micro-grids schedule their resources to operate when the buying price is high or when their suggested quantity is not available in the transactive market. Connected entities to the upper grid buy from electricity markets at low prices and sells to the micro-grids in transactive market by implementing real time forecasts.

## Appendix A

ADMM is an distributed algorithm that is introduced to consider the decomposability of dual ascent with taken into account convergence properties. The algorithm can solve problems with complicated constraint which are in the form:

$$\begin{aligned} \min \quad & f(x) + g(y) \\ \text{s.t.} \quad & Ax + By - C = 0 \quad : \lambda \end{aligned} \quad (\text{A.1})$$

According the proofed represented in [26], the necessary and sufficient optimality conditions for the ADMM problem (A1) are primal feasibility:

$$0 = \nabla f(x^*) + A^T \lambda^* \quad (\text{A.2})$$

and dual feasibility:

$$0 = \nabla g(y^*) + B^T \lambda^* \quad (\text{A.3})$$

It is proved in [26], that two residuals related to the primal variables (primal residual) and related to dual variable (dual residual) can be introduced which should be considered as termination criteria:

$$r^{k+1} = Ax^{k+1} + By^{k+1} - C \quad (\text{A.4})$$

$$s^{k+1} = \rho A^T B(y^{k+1} - y^k) \quad (\text{A.5})$$

The termination condition can be considered as:

$$\|r^k\|_2 = \epsilon^{pri} \quad (\text{A.6})$$

$$\|s^k\|_2 = \epsilon^{dual} \quad (\text{A.7})$$

where  $\epsilon^{pri} > 0$  and  $\epsilon^{dual} > 0$  are feasibility tolerances for the primal and dual feasibility conditions (A.4) and (A.5), respectively. It is suggested that tolerances be calculated as follow:

$$\epsilon^{pri} = \sqrt{\rho} \epsilon^{abs} + \epsilon^{rel} \max\{\|Ax^k\|_2, \|By^k\|_2, \|C\|\} \quad (\text{A.8})$$

$$\epsilon^{dual} = \sqrt{\rho} \epsilon^{abs} + \epsilon^{rel} \max\{\|A^T B y^k\|_2\} \quad (\text{A.9})$$

where  $\epsilon^{abs} > 0$  and  $\epsilon^{relative} > 0$  are positive absolute and relative tolerances.

## References

- [1] Hatzigaryriou ND, Klefakis VA, Papadimitriou CN, Messinis GM. Microgrids in distribution. Smart Grid Handbook 2016:1–24.
- [2] Hammerstrom DJ, Widergren SE, Irwin C. Evaluating transactive systems: Historical and current us doe research and development activities. IEEE Electrif Mag 2016;4(4):30–6.
- [3] Kristov L, De Martini P, Taft JD. A tale of two visions: Designing a decentralized transactive electric system. IEEE Power Energ Mag 2016;14(3):63–9.

Distributing the role of proposed IEE between players to handle double auction markets needs to be studied in the future works.

## CRedit authorship contribution statement

**Mehdi Jalali:** Conceptualization, Methodology, Writing – original draft. **Kazem Zare:** Supervision, Project administration, Writing – review & editing. **Sajjad Tohidi:** Writing – review & editing, Validation, Formal analysis.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Acknowledgement

This work was supported by a research grant of the University of Tabriz under Grant S/4345.

- [4] Qiu J, Meng K, Zheng Y, Dong ZY. Optimal scheduling of distributed energy resources as a virtual power plant in a transactive energy framework. *IET Generat Transmiss Distrib* 2017;11(13):3417–27.
- [5] Sajjadi SM, Mandal P, Tseng T-LB, Velez-Reyes M. Transactive energy market in distribution systems: A case study of energy trading between transactive nodes. In: *North American Power Symposium (NAPS)*, 2016. IEEE; 2016. p. 1–6.
- [6] Renani YK, Ehsan M, Shahidehpour M. Optimal transactive market operations with distribution system operators. *IEEE Trans Smart Grid* 2018;9(6):6692–701.
- [7] Akter MN, Mahmud MA, Oo AM. An optimal distributed transactive energy sharing approach for residential microgrids. In: *Power & Energy Society General Meeting, 2017 IEEE*. IEEE; 2017. p. 1–5.
- [8] Jalali M, Zare K, Seyedi H. Strategic decision-making of distribution network operator with multi-microgrids considering demand response program. *Energy* 2017;141:1059–71.
- [9] Kou P, Liang D, Gao L. Distributed empc of multiple microgrids for coordinated stochastic energy management. *Appl Energy* 2017;185:939–52.
- [10] Wu J, Guan X. Coordinated multi-microgrids optimal control algorithm for smart distribution management system. *IEEE Trans Smart Grid* 2013;4(4):2174–81.
- [11] Shi W, Xie X, Chu C-CP, Gadh R. Distributed optimal energy management in microgrids. *IEEE Trans Smart Grid* 2015;6(3):1137–46.
- [12] Hug G, Kar S, Wu C. Consensus+ innovations approach for distributed multiagent coordination in a microgrid. *IEEE Trans Smart Grid* 2015;6(4):1893–903.
- [13] Zheng Y, Song Y, Hill DJ, Zhang Y. Multiagent system based microgrid energy management via asynchronous consensus admm. *IEEE Trans Energy Convers* 2018; 33(2):886–8.
- [14] Jalali M, Zare K, Seyedi H, Alipour M, Wang F. Distributed model for robust real-time operation of distribution systems and microgrids. *Electric Power Syst Res* 2019;177:105985.
- [15] Gao H, Liu J, Wang L, Wei Z. Decentralized energy management for networked microgrids in future distribution systems. *IEEE Trans Power Syst* 2018;33(4): 3599–610.
- [16] Soares AR, De Somer O, Ectors D, Aben F, Goyvaerts J, Broekmans M, Spiessens F, van Goch D, Vanthournout K. Distributed optimization algorithm for residential flexibility activation—results from a field test. *IEEE Trans Power Syst* 2018.
- [17] Morstyn T, Teytelboym A, McCulloch MD. Bilateral contract networks for peer-to-peer energy trading. *IEEE Trans Smart Grid* 2018;10(2):2026–35.
- [18] Guerrero J, Chapman AC, Verbić G. Decentralized p2p energy trading under network constraints in a low-voltage network. *IEEE Trans Smart Grid* 2018.
- [19] Morstyn T, McCulloch M. Multi-class energy management for peer-to-peer energy trading driven by prosumer preferences. *IEEE Trans Power Syst* 2018.
- [20] Hu J, Yang G, Ziras C, Kok K. Aggregator operation in the balancing market through network-constrained transactive energy. *IEEE Trans Power Syst* 2018.
- [21] Moslehi K, Kumar R. Autonomous resilient grids in an iot landscape—vision for a nested transactive grid. *IEEE Trans Power Syst* 2018.
- [22] Moret F, Pinson P. Energy collectives: a community and fairness based approach to future electricity markets. *IEEE Trans Power Syst* 2018.
- [23] Liu W, Zhan J, Chung C. A novel transactive energy control mechanism for collaborative networked microgrids. *IEEE Trans Power Syst* 2018;34(3):2048–60.
- [24] Nazir MS, Hiskens IA. A dynamical systems approach to modeling and analysis of transactive energy coordination. *IEEE Trans Power Syst* 2018.
- [25] Lian J, Ren H, Sun Y, Hammerstrom D. Performance evaluation for transactive energy systems using double-auction market. *IEEE Trans Power Syst* 2018.
- [26] Boyd S, Parikh N, Chu E, Peleato B, Eckstein J, et al. Distributed optimization and statistical learning via the alternating direction method of multipliers. *Found Trends Machine Learn* 2011;3(1):1–122.
- [27] Khodayar ME, Barati M, Shahidehpour M. Integration of high reliability distribution system in microgrid operation. *IEEE Trans Smart Grid* 2012;3(4): 1997–2006.
- [28] Boyd S. Alternating direction method of multipliers. In: *Talk at NIPS workshop on optimization and machine learning*, 2011.
- [29] <https://github.com/mahdijalalipower/mehdi-jalali.git>.
- [30] Arteaga J, Zareipour H. A price-maker/price-taker model for the operation of battery storage systems in electricity markets. *IEEE Trans Smart Grid* 2019;10(6): 6912–20.